SAT Math Must-Know Facts & Formulas

Numbers, Sequences, Factors

Integers: . . . , -3, -2, -1, 0, 1, 2, 3, . . .

Rationals: fractions, that is, anything expressable as a ratio of integers

Reals: integers plus rationals plus special numbers such as $\sqrt{2}$, $\sqrt{3}$ and π

Order Of Operations: PEMDAS
(Parentheses / Exponents / Multiply / Divide / Add / Subtract)

Arithmetic Sequences: each term is equal to the previous term plus d

Sequence: t_1, $t_1 + d$, $t_1 + 2d$, . . .

Example: $d = 4$ and $t_1 = 3$ gives the sequence 3, 7, 11, 15, . . .

Geometric Sequences: each term is equal to the previous term times r

Sequence: t_1, $t_1 \cdot r$, $t_1 \cdot r^2$, . . .

Example: $r = 2$ and $t_1 = 3$ gives the sequence 3, 6, 12, 24, . . .

Factors: the factors of a number divide into that number without a remainder

Example: the factors of 52 are 1, 2, 4, 13, 26, and 52

Multiples: the multiples of a number are divisible by that number without a remainder

Example: the positive multiples of 20 are 20, 40, 60, 80, . . .

Percents: use the following formula to find part, whole, or percent

$$\text{part} = \frac{\text{percent}}{100} \times \text{whole}$$

Example: 75% of 300 is what?
Solve $x = (75/100) \times 300$ to get 225

Example: 45 is what percent of 60?
Solve $45 = (x/100) \times 60$ to get 75%

Example: 30 is 20% of what?
Solve $30 = (20/100) \times x$ to get 150

SAT Math Must-Know Facts & Formulas

Averages, Counting, Statistics, Probability

$$\text{average} = \frac{\text{sum of terms}}{\text{number of terms}}$$

$$\text{average speed} = \frac{\text{total distance}}{\text{total time}}$$

$$\text{sum} = \text{average} \times (\text{number of terms})$$

$$\text{mode} = \text{value in the list that appears most often}$$

$$\text{median} = \text{middle value in the list (which } \textit{must} \text{ be sorted)}$$

Example: median of $\{3, 10, 9, 27, 50\} = 10$
Example: median of $\{3, 9, 10, 27\} = (9 + 10)/2 = 9.5$

Fundamental Counting Principle:

If an event can happen in N ways, and another, independent event can happen in M ways, then both events together can happen in $N \times M$ ways.

Probability:

$$\text{probability} = \frac{\text{number of desired outcomes}}{\text{number of total outcomes}}$$

Example: each SAT math multiple choice question has five possible answers, one of which is the correct answer. If you guess the answer to a question completely at random, your probability of getting it right is $1/5 = 20\%$.

The probability of two different events A and B *both* happening is $P(A \text{ and } B) = P(A) \cdot P(B)$, as long as the events are independent (not mutually exclusive).

Powers, Exponents, Roots

$$x^a \cdot x^b = x^{a+b} \qquad x^a/x^b = x^{a-b} \qquad 1/x^b = x^{-b}$$

$$(x^a)^b = x^{a \cdot b} \qquad (xy)^a = x^a \cdot y^a \qquad (-1)^n = \begin{cases} +1, & \text{if } n \text{ is even;} \\ -1, & \text{if } n \text{ is odd.} \end{cases}$$

$$x^0 = 1 \qquad \sqrt{xy} = \sqrt{x} \cdot \sqrt{y}$$

Factoring, Solving

$$(x+a)(x+b) = x^2 + (b+a)x + ab \qquad \text{“FOIL”}$$

$$a^2 - b^2 = (a+b)(a-b) \qquad \text{“Difference Of Squares”}$$

$$a^2 + 2ab + b^2 = (a+b)(a+b)$$
$$a^2 - 2ab + b^2 = (a-b)(a-b)$$

To solve a quadratic such as $x^2+bx+c=0$, first factor the left side to get $(x+a_1)(x+a_2)=0$, then set each part in parentheses equal to zero. E.g., $x^2+4x+3=(x+3)(x+1)=0$ so that $x=-3$ or $x=-1$.

To solve two linear equations in x and y: use the first equation to substitute for a variable in the second. E.g., suppose $x+y=3$ and $4x-y=2$. The first equation gives $y=3-x$, so the second equation becomes $4x-(3-x)=2 \Rightarrow 5x-3=2 \Rightarrow x=1, y=2$.

Functions

A function is a rule to go from one number (x) to another number (y), usually written

$$y = f(x).$$

For any given value of x, there can only be one corresponding value y. If $y=kx$ for some number k (example: $f(x)=0.5\cdot x$), then y is said to be *directly proportional* to x. If $y=k/x$ (example: $f(x)=5/x$), then y is said to be *inversely proportional* to x.

Absolute value:

$$|x| = \begin{cases} +x, & \text{if } x \geq 0; \\ -x, & \text{if } x < 0. \end{cases}$$

Lines (Linear Functions)

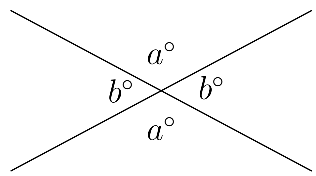
Consider the line that goes through points $A(x_1, y_1)$ and $B(x_2, y_2)$.

Distance from A to B: $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

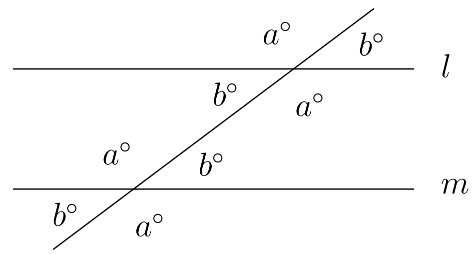
Mid-point of the segment $\overline{AB}$: $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

Slope of the line: $\frac{y_2-y_1}{x_2-x_1} = \frac{\text{rise}}{\text{run}}$

Slope-intercept form: given the slope m and the y-intercept b, then the equation of the line is $y = mx + b$. Parallel lines have equal slopes: $m_1 = m_2$. Perpendicular lines have negative reciprocal slopes: $m_1 \cdot m_2 = -1$.



Intersecting Lines



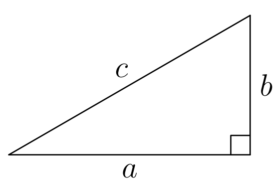
Parallel Lines ($l \parallel m$)

Intersecting lines: opposite angles are equal. Also, each pair of angles along the same line add to 180°. In the figure above, $a + b = 180°$.

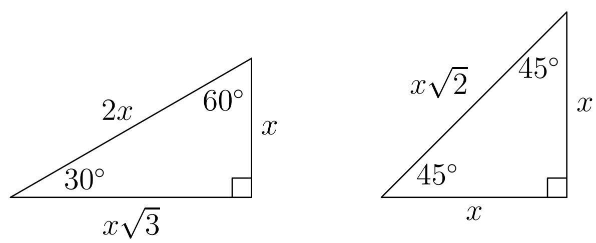
Parallel lines: eight angles are formed when a line crosses two parallel lines. The four big angles (a) are equal, and the four small angles (b) are equal.

Triangles

Right triangles:



$a^2 + b^2 = c^2$



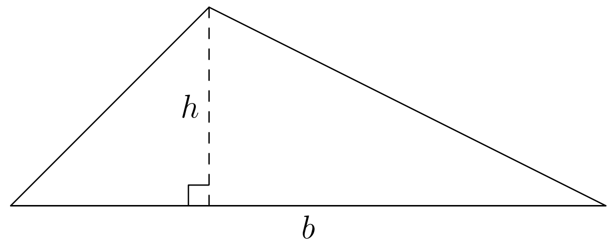
Special Right Triangles

Note that the above special triangle figures are given in the test booklet, so you don't have to memorize them, but you should be familiar with what they mean, especially the first one, which is called the Pythagorean Theorem ($a^2 + b^2 = c^2$).

A good example of a right triangle is one with $a = 3$, $b = 4$, and $c = 5$, also called a 3–4–5 right triangle. Note that multiples of these numbers are also right triangles. For example, if you multiply these numbers by 2, you get $a = 6$, $b = 8$, and $c = 10$ (6–8–10), which is also a right triangle.

The "Special Right Triangles" are needed less often than the Pythagorean Theorem. Here, "x" is used to mean any positive number, such as 1, 1/2, etc. A typical example on the test: you are given a triangle with sides 2, 1, and $\sqrt{3}$ and are asked for the angle opposite the $\sqrt{3}$. The figure shows that this angle is 60°.

All triangles:



$$\text{Area} = \frac{1}{2} \cdot b \cdot h$$

The area formula above works for *all* triangles, not just right triangles.

Angles on the inside of any triangle add up to 180°.

The length of one side of any triangle is always *less* than the sum of the lengths of the other two sides.

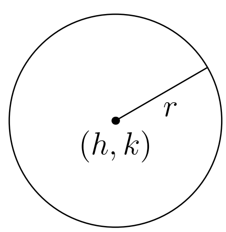
Other important triangles:

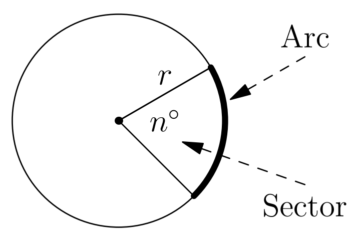
Equilateral: These triangles have three equal sides, and all three angles are 60°.

Isosceles: An isosceles triangle has two equal sides. The "base" angles (the ones opposite the two sides) are equal. A good example of an isosceles triangle is the one on page 4 with base angles of 45°.

Similar: Two or more triangles are similar if they have the same shape. The corresponding angles are equal, and the corresponding sides are in proportion. For example, the 3–4–5 triangle and the 6–8–10 triangle from before are similar since their sides are in a ratio of 2 to 1.

Circles



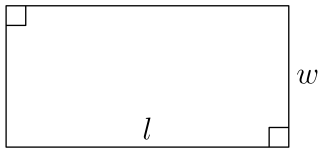


$$\text{Area} = \pi r^2$$
$$\text{Circumference} = 2\pi r$$
$$\text{Full circle} = 360^\circ$$

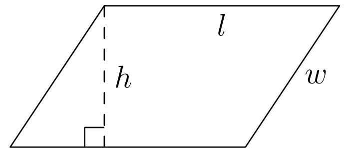
(Optional)

$$\text{Length Of Arc} = (n^\circ/360^\circ) \cdot 2\pi r$$
$$\text{Area Of Sector} = (n^\circ/360^\circ) \cdot \pi r^2$$

Rectangles And Friends



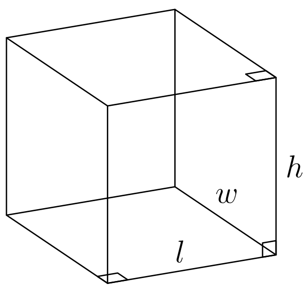
Rectangle
(Square if $l = w$)
Area $= lw$



Parallelogram (Optional)
(Rhombus if $l = w$)
Area $= lh$

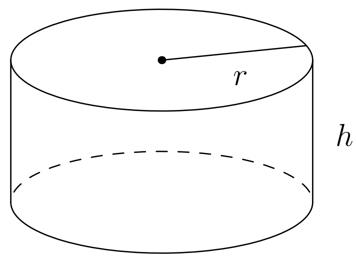
The formula for the area of a rectangle is given in the test booklet, but it is very important to know, so you should memorize it anyway.

Solids



Rectangular Solid

Volume $= lwh$



Right Cylinder

Volume $= \pi r^2 h$

Note that the above solids figures are given in the test booklet, so you don't have to memorize them, but you should be familiar with what they mean.